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PROBABILISTIC BASELINE GENERATOR FOR ROLE
DIFFERENTIATION IN FORMAL ORGANIZATIONS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Algorithms are suggested for generating probabilistic distributions of organization role differentiation for given organization sizes under five general assumptions. The assumptions deal with uniqueness among the skills of organization members and among skill demands of organization roles. The algorithms calculate the distributions of all logically possible arrange- ments of <u>s</u> people filling <u>k</u> roles. This procedure yields a percentile role differentiation indicator (PRDI) for any given organization size <u>s</u> under		

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each of the five general assumptions. A PRDI value indicates how much of an organization's empirical role differentiation is accounted for by size, allowing an investigator to examine the effects of other influences upon complexity. A series of PRDI values across time tell the effects of other variables besides size upon the complexity of a growing organization. In cross-sectional analysis, PRDI values eliminate the logical effects of size alone upon the complexity of a set of organizations of different sizes.

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1. INTRODUCTION.

Through the popularity of C. Northcote Parkinson, whose tongue-in-cheek words invariably become Law, virtually the entire civilized world knows that as organizations grow they also become more bureaucratic. The less pedestrian statement of this notion is that organization complexity is a function of organization size.¹

Blau (1970) has been among the first contemporary students of organization growth to concisely model the relationship between size and structural differentiation. He suggests that (a) differentiation is a monotone increasing function of size while (b) the rate of increase in differentiation is a monotone decreasing function of size.

Blau theorized inductively from a considerable store of data that he and his associates gathered from governmental units (Blau and Schoenherr, 1971). Mayhew, McPherson, Levinger, and James (1972) have alternatively tried to deductively predict values for organization differentiation from organization size. They offer an algorithm that generates all logically possible values of differentiation for varying sizes, assigning occurrence probabilities to each possible arrangement of a certain number of people into a set of organization roles. They held constant all other variables that might influence variation in role structure in order to compute the logical effects of size alone.

¹In sociology literature, complexity is often synonymous with differentiation in order to emphasize the intensity of specialization or division of labor that is possible in large, formal organizations. Differentiation could be measured according to the number of sub-units with an organization or by the number of its levels of authority. However, by convention, complexity or differentiation means the number of job roles in an organization.

Size, too, has a variety of possible definitions but the usual definition is the total number of employees or members in an organization.

Mayhew et al. did not make explicit assumptions about the interchangeability of pairs of people assigned to organizations roles. Nor did they make assumptions about inherent distinctions between the roles themselves. They did imply the traditional authority pyramid in which the most populous roles are assumed to be at the bottom of an organization.

In order to generalize the deduction of size-differentiation models, we offer algorithms for estimating values of differentiation (number of roles in an organization) for given sizes under different assumptions about (a) the interchangeability of the roles themselves and (b) about the interchangeability of organization members between roles.

A role is defined here as a set of people doing the same work such that the persons within a role have no hierarchical distinctions among themselves. We assume that members of a role are freely interchangeable within that role without affecting the structure of the organization. This means that, in Figure 1, if member 2 and member 3 switched places within Role A of the overall ABC organization, it would not count as a logically possible, genuine new arrangement of the 10 members and three roles.

Throughout the following discussion of the different assumptions about role and people arrangements, the references to interchangeability of individual members between roles assumes that such interchanges are limited to one interchange at a time. We will, for example, make assumptions about the effect of a switch between member 3 in Role A and member 4 in Role B. We will not, however, consider a simultaneous switch between members 1 and 4, 2 and 5, 3 and 6. An interchange of all members between two roles would usually modify the structure of an organization. In effect, it would be a switching of roles themselves in the workflow scheme. One man roles could not, by definition, be party to any interchanges except as roles.

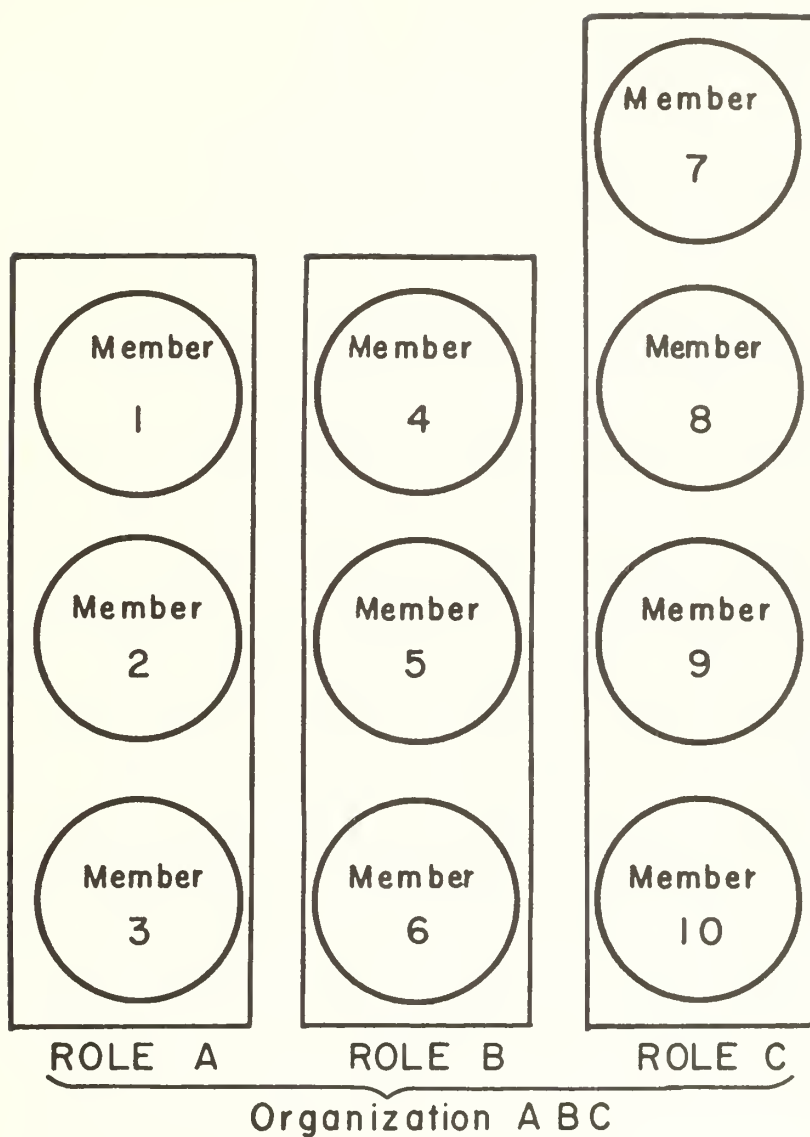


FIGURE 1. Illustration of a hypothetical small organization of size $\underline{s} = 10$ and \underline{k} roles = 3 in one of \underline{n} logically possible arrangements.

The five interchangeability assumptions we present here are not the only ones possible, but they are the most general. One could assume, for example, that a particular kind of role, say, clerical staff, is to be analyzed as an element of administrative overhead. Clerical roles could be treated as non-interchangeable (defined below) with the other roles in an organization regardless of what assumption was made about the interchangeability of the latter among themselves. A special formulation of the theoretical possibilities of clerical role differentiation for changing organization size could be developed for such an analysis.

2. DEFINITIONS.

To say that individual members of an organization are non-interchangeable is to say that if two members of different roles switched places, then a legitimate new arrangement of the organization would occur. In the algebraic notation used here, it would mean that such an interchange of people would be counted as another way of arranging s people in k roles, adding to the number of such logically possible arrangements, n .

Conversely, people in an organization are interchangeable if two members can switch places between roles and it does not result in a legitimate new arrangement of an organization.

The assignment of people to roles is denoted by r_j , the number of people in role j . Roles in an organization are non-interchangeable if they are viewed as having some order, whether actual or arbitrarily assigned, among themselves. For example, one organization arrangement might consist of Role A followed by Role B which, in turn, was followed by Role C. If the order were reversed to C-B-A and an organization was thereby considered to be of a new legitimate form, then those three roles would be treated as non-interchangeable. The order of the roles would have affected the number of possible organizational forms for a given size s .

Where the order of relationship among roles does not count as a different arrangement, or forms an illegitimate arrangement, then the roles are interchangeable. Under that assumption, both orders, A-B-C and C-B-A , would be considered as but one arrangement.

3. ASSUMPTIONS.

3.1 Assumption I: Roles Are Not Interchangeable/People Are Not Interchangeable

Under this assumption, any changing of places by individuals between roles, or changing of places by roles in a work scheme, would be considered a new legitimate organization arrangement. In Figure 1, if member 4 switched places with member 1 then an organization would be considered to have been materially altered in structure. The number of logically possible arrangements of s people would be increased by one. The same with roles. If Role A and Role B were interchanged in a workflow, it would constitute a new organization and again the number of possible arrangements would rise by one. The same would happen if Role B and Role C were switched, even though they are of different sizes.

Assumption I is not realistic for the traditional Weberian (Weber, 1947) authority hierarchy, or what is popularly known as the bureaucratic form of organization. An example can be imagined, however, in which organization ABC (Figure 1) were a newspaper office with three news editors (Role A), three copy editors (Role B), and four reporters (Role C). If a news editor (#1) switched places with a copy editor (#4), then organization ABC would be considered to have a new form. Likewise, if all the copy editors (Role B) changed places with all of the reporters (Role C) in a daily newspaper workflow, the organization would again be considered to have a new arrangement.

If \underline{s} non-interchangeable persons are to be assigned to \underline{k} non-interchangeable roles, let \underline{r}_j , $j = 1 \dots \underline{k}$, be the number of people in role \underline{j} and let \underline{n}_i , $i = 1 \dots \underline{s}$, be the number of roles with \underline{i} people assigned to them. Thus,

$$\sum_{j=1}^{\underline{k}} \underline{r}_j = \sum_{i=1}^{\underline{s}} \underline{i} \underline{n}_i = \underline{s} \quad \text{and} \quad \sum_{i=1}^{\underline{s}} \underline{n}_i = \underline{k}.$$

An assignment of non-interchangeable persons to non-interchangeable roles is therefore a sequence $(\underline{r}_1 \dots \underline{r}_{\underline{k}})$ of \underline{k} positive integers which sum to \underline{s} . The number of logically possible different ways of assigning \underline{s} non-interchangeable persons to \underline{k} non-interchangeable roles is

$$\frac{\underline{s}!}{\prod_{i=1}^{\underline{s}} (\underline{i}!)^{\underline{n}_i}} = \frac{\underline{s}!}{\prod_{j=1}^{\underline{k}} (\underline{r}_j!)}.$$

Summing across all possible assignments, there are $\underline{k}! S_{\underline{s}}^{(\underline{k})}$ different arrangements of non-interchangeable people in non-interchangeable roles for an organization of size \underline{s} . The value of $S_{\underline{s}}^{(\underline{k})}$ will be discussed in Assumption II.

3.2 Assumption II: Same Size Roles Are Interchangeable / People Are Not Interchangeable.

Under this assumption, the order of roles among themselves is not relevant to counting new arrangements of an organization but distinctions are made about order between roles of different sizes. Role A (Figure 1) could be switched with Role B in a workflow scheme and it would not be considered that organization ABC had taken a legitimately new form. However, if either Role A or Role B were interchanged with the larger Role C a new organization arrangement would be counted as an addition to \underline{n} .

The treatment of interchanges between individual members in different roles is the same as under assumption I.

Assumption II is approximated in practice by the treatment of day shifts in a production operation. If both shifts are of the same size (as between Role A and Role B in Figure 1), then interchanging them has little effect on the basic form of an organization. However, if the night shift is essentially a skeleton crew (Role B) while the day shift is staffed for a heavier workload (Role C), then interchanging them would upset the essential structure of a workflow.

If \underline{s} non-interchangeable persons are to be assigned to \underline{k} roles where roles of the same size are interchangeable, then the number of logically possible different ways of assigning them decreases to

$$\frac{\underline{s}!}{\prod_{\underline{i}=1}^{\underline{s}} (\underline{i}!)^{\underline{n}_{\underline{i}}} \underline{n}_{\underline{i}}!} = \frac{\underline{s}!}{\prod_{\underline{j}=1}^{\underline{k}} \underline{r}_{\underline{j}}! \prod_{\underline{i}=1}^{\underline{s}} \underline{n}_{\underline{i}}!} .$$

Summing across all possible assignments yields $S_{\underline{s}}^{(\underline{k})}$ different ways of assigning \underline{s} non-interchangeable people to \underline{k} roles such that only roles of equal size are interchangeable. $S_{\underline{s}}^{(\underline{k})}$ is the Stirling number of the second type;

$$S_{\underline{s}}^{(\underline{k})} = \frac{1}{\underline{k}!} \sum_{\underline{j}=1}^{\underline{k}} (-1)^{\underline{k}-\underline{j}} \binom{\underline{k}}{\underline{j}} \underline{j}^{\underline{s}}$$

The calculation of $S_{\underline{s}}^{(\underline{k})}$ can be facilitated by the following recurrence relations (See Moser and Wyman, 1958):

$$(1) \quad S_{\underline{s}}^{(1)} = S_{\underline{s}}^{(\underline{s})} = 1$$

$$(2) \quad S_{\underline{s}}^{(\underline{s}-1)} = \binom{\underline{s}}{2}$$

$$(3) \quad S_{\underline{s}+1}^{(\underline{k})} = \underline{k} S_{\underline{s}}^{(\underline{k})} + S_{\underline{s}}^{(\underline{k}-1)}$$

3.3 Assumption III: Roles Are Not Interchangeable / People Are Interchangeable

In this situation, as under assumption I, any switching of roles in a workflow scheme, regardless of role size, will constitute a new form of an organization. However, any two individual members could switch jobs without such an interchange affecting an organization under this assumption. Going back to Figure 1 again, any of the members, from #1 to #10, would be assumed capable of handling any job in organization ABC. Assumption III is approximated in reality by small organizations that cross-train their members to handle any job in the workflow with some minimum competence. Some units of the U.S. Air Force adhere to this principle. An imaginary example would be an office (organization ABC) staffed with three typists (Role A), three receptionists (Role B), and four executive secretaries (Role C). The roles would be fairly distinct as to their respective contributions to the operation of the office. They could not be readily switched about. Yet, the people working in any of the three roles might be minimally capable of working in the others. This is the case in small offices.

If the people available to fill jobs in an organization are assumed to be interchangeable, then the ways of arranging them in an organization of size \underline{s} would involve only the hierarchical distinctions among roles. The number of logical ways of assigning them to roles would be $\underline{k}!$. Summing over all possible arrangements of \underline{s} persons in \underline{k} roles would result in $\underline{k}! N_{\underline{s}}^{(\underline{k})}$ ways of assigning them. The calculation of $N_{\underline{s}}^{(\underline{k})}$ will be discussed under assumption V.

3.4 Assumption IV: Same Size Roles Are Interchangeable / People Are Interchangeable.

Here, as with assumption II, roles of the same size can be switched in a workflow scheme without considering the interchange to be a new legitimate organization arrangement. But a switch between roles of different sizes would be

counted as a new arrangement. Unlike assumption II, however, the relationship of individuals is such that they can switch places between roles without creating a new legitimate organization arrangement.

The shift work example is closer to conventional management practice under this assumption. Consider organization ABC (Figure 1 again) as a hospital ward. Role C could be four nurses on the day shift, Role A could be three nurses on the afternoon shift and Role B could be three on the night shift. Role A and Role B are readily interchangeable but neither can easily switch places with Role C because the difference in size would constitute a new organization arrangement. But any one of the ten nurses could change places with any of the others on a different shift without altering the count of logically possible arrangements under this assumption.

Under these circumstances the number of logically possible ways of assigning \underline{s} people to \underline{k} roles is

$$\frac{\underline{k}!}{\prod_{\underline{i}=1}^{\underline{s}} \underline{n_i}!} .$$

Summing over all possible arrangements of people in roles under this assumption leads to

$$\binom{\underline{s}-1}{\underline{k}-1} = \frac{(\underline{s}-1)!}{(\underline{k}-1)!(\underline{s}-\underline{k})!} \quad \text{ways of making assignments. The}$$

recurrence relations for $\frac{\underline{s}}{\underline{k}}$ are:

$$(1) \quad \binom{\underline{s}}{0} = 1$$

$$(2) \quad \binom{\underline{s}}{\underline{s}} = 1$$

$$(3) \quad \binom{\underline{s}+1}{\underline{k}} = \binom{\underline{s}}{\underline{k}-1} + \binom{\underline{s}}{\underline{k}}$$

3.4 Assumption V: Roles Are Interchangeable/People Are Interchangeable.

Finally, we allow that any two roles may switch places in a workflow scheme regardless of their size, without bringing a new legitimate organization arrangement into being. The same interchangeability of individual members applies here as it did under assumptions III and IV. This situation comes closest to the traditional authority hierarchy that is the usual format of complex organizations. Think about staffing an organization from scratch. You would have \underline{k} roles to be filled with \underline{s} available manpower. You would, if adhering to the Weberian model of bureaucracy (Weber, 1947), match the skills among the \underline{s} people with the skill and experience requirements of the \underline{k} roles as best you could. Once the roles were filled and the organization running you would not allow people to switch to roles for which they did not have the minimum competence. A janitor in a school would not ordinarily switch jobs with a teacher. Such a switch therefore would not be "allowable" under this assumption in terms of its effect on the counting of \underline{n} logically possible organization arrangements.

An example of the application of assumption V would be if organization ABC (once again to Figure 1) were an academic department of operations research staffed by three mathematicians (Role A), three statisticians (Role B), and four operations analysts (Role C). Should one operations analyst decide that he was actually a statistician, the resulting structure would be three mathematicians, four statisticians, and three operations analysts. Under this assumption, both arrangements of the ten members in the three roles would be considered as one in computing \underline{n} .

By making both individuals and roles interchangeable the sequence $(\underline{r}_1, \dots, \underline{r}_{\underline{k}})$ will be restricted to a non-increasing order. The number of non-increasing order arrangements of length \underline{k} that sum to \underline{s} are represented by $N_{\underline{s}}^{(\underline{k})}$. The generating function for $N_{\underline{s}}^{(\underline{k})}$ is

$$p(t, \underline{k}) = \sum_{\underline{s}=1}^{\infty} N_{\underline{s}}^{(\underline{k})} t^{\underline{s}} = \frac{t^{\underline{k}}}{\prod_{\underline{i}=1}^{\underline{k}} (1-t^{\underline{i}})} .$$

Analysis of $p(t, \underline{k})$ yields the following²:

- (1) $N_{\underline{s}}^{(\underline{k})} = 0$ if $\underline{s} < \underline{k}$
- (2) $N_{\underline{s}}^{(1)} = 1$ if $\underline{s} = 1, 2, \dots$
- (3) $N_{\underline{s}}^{(\underline{k})} = N_{\underline{s}-1}^{(\underline{k}-1)} + N_{\underline{s}-\underline{k}}^{(\underline{k})}$ if $\underline{k} \leq \frac{\underline{s}}{2}$
- (4) $\sum_{\underline{j}=1}^{\underline{k}} N_{\underline{s}}^{(\underline{j})} = N_{\underline{s}+\underline{k}}^{(\underline{k})}$
- (5) $N_{\underline{s}}^{(\underline{k})} = N_{2\underline{s}-2\underline{k}}^{(\underline{s}-\underline{k})}$ if $\underline{k} > \frac{\underline{s}}{2}$

Results (3) and (5) above are recurrence relations that are useful in generating a probability table for $N_{\underline{s}}^{(\underline{k})}$.

By noting that $N_{2\underline{s}}^{(\underline{s})} = \sum_{\underline{j}=1}^{\underline{s}} N_{\underline{s}}^{(\underline{j})} = p_{\underline{s}}$ the generating function for $p_{\underline{s}}$ is obtained as

$$p(t) = \sum_{\underline{s}=1}^{\infty} p_{\underline{s}} t^{\underline{s}} + 1 = \sum_{\underline{k}=1}^{\infty} p(t, \underline{k}) = \frac{1}{\prod_{\underline{i}=1}^{\infty} (1-t^{\underline{i}})}$$

This generating function has been used by Gupta (1935, 1937) to generate a table of $p_{\underline{s}}$ and to demonstrate that

$$N_{2\underline{s}}^{(\underline{s})} \approx \frac{1}{\sqrt{48\underline{s}}} e^{\pi \frac{2\underline{s}}{3}}$$

²A helpful explanation of this analysis is found in Riordan (1958) Chapter 6.

for organizations of large size. In general

$$N_{\underline{s}}^{(\underline{k})} = \left[\frac{\underline{k}(\underline{s}-\underline{k})}{(\underline{k}!)^2} u_{\underline{k}-2}^{(\underline{s})} \right] + 1$$

where the braces denote the integer part of the value enclosed and $u_L^{(\underline{s})}$ is an L th order polynomial in \underline{s} . For example³, $u_0(\underline{s}) = 1$, $u_1(\underline{s}) = \underline{s} + 3$, and $u_2(\underline{s}) = \underline{s}^2 + 7\underline{s} + 28$.

4. APPLICATIONS.

4.1 Application I: Measure of Differentiation.

Suppose an organization employing \underline{s} people is being studied which assigns these people to work roles according to any one of the preceding general assumptions. Let $P(\underline{k}, \underline{s}) = P[K = \underline{k} | \underline{s}]$ be the probability of assigning \underline{s} people to \underline{k} roles under that assumption. By assuming every assignment is equally likely, then $P(\underline{k}, \underline{s})$ is the number of ways of assigning \underline{s} people to \underline{k} roles divided by the total number of assignments under the particular assumption.

One way of assessing the structural differentiation of a given organization with \underline{k} roles is by noting the probability that a randomly chosen organization of size \underline{s} could have fewer than \underline{k} roles. Thus,

$$M_1(\underline{k}, \underline{s}) = \sum_{\underline{i}=1}^{\underline{k}-1} P(\underline{i}, \underline{s}) = P[K < \underline{k} | \underline{s}]$$

where $M_1(\underline{k}, \underline{s})$ is the percentile indicator of differentiation ranging from 0% to 100%. 0% would indicate that an organization contained but a single role. This would represent a polar case of structural simplicity, being non-differentiation. On the other hand, 100% would signify that there were

³Tables of $N_{\underline{s}}^{(\underline{k})}$ for $s=1$ to 2000 are available upon request from W.J. Haga, Department of Operations Research and Administrative Science, Naval Postgraduate School, Monterey, California 93940.

more roles than there were individual members in the organization. This would be a polar case of complexity.

Another means of gauging differentiation in organization role structure is to look at the distance, in probability, that the actual number of roles comprising an organization is from the mean expected number of roles for an organization of that size according to the distribution of all logical possibilities. Let

$$\bar{K} = \sum_{\underline{i}=1}^{\underline{s}} \underline{i}P(\underline{i},\underline{s}) \quad \text{and}$$

$$\begin{aligned} M_2(\underline{k},\underline{s}) &= P[|K-\bar{K}| < |\underline{k}-\bar{K}|] \\ &= M_1(\bar{K} + |\bar{K}-\underline{k}|,\underline{s}) - M_1(\bar{K} - |\bar{K}-\underline{k}| + 1,\underline{s}) \quad . \end{aligned}$$

The size of an organization has been observed to be positively correlated to number of roles. The formulation of M_1 is such that, if the organization does not change its role differentiation assumption and maintains the same tendency towards complexity, the measure should not change with size. In reality some small changes occur because of the discrete nature of the process; and these changes will be minimal for large organizations. Thus, for large organizations the percentile of complexity is size invariant. Among organizations of different sizes, their complexity can be compared according to their relative percentile indicators of logically possible role differentiation.

Any change in M_1 can either be attributed to a change in complexity or a deviance from the assumption utilized to calculate M_1 . The expected value of M_1 for large organizations is 50%. Depending on the assumption chosen, the percentile of the expected number of roles \bar{K} may be more than or less

than .5 depending upon the tendency of that assumption to produce more complex or less complex organizations. A measure of this tendency of an assumption to or away from complexity is

$$C = \frac{M_1(\bar{k}, \underline{s}) - .5}{.5}$$

which ranges from -1 for the least complex to +1 for the most complex.

4.2 Application II: Deviations From Predicted Differentiation.

Suppose several, \underline{n} , organizations of approximately the same size, \underline{s} , appear to adhere to one of the general assumptions. Let $\hat{p}(\underline{k}, \underline{s})$ be the proportion of organizations of \underline{s} number of people with \underline{k} number of roles. How well this sample of organizations fit the probabilistic model of role differentiation according to any general assumption is

$$\chi^2 = \underline{n} \sum_{\underline{k}=1}^{\underline{s}} \frac{(\hat{p}(\underline{k}, \underline{s}) - P(\underline{k}, \underline{s}))^2}{P(\underline{k}, \underline{s})}$$

which is a chi-squared random variable with $\underline{s}-1$ degrees of freedom.

Two reasons for yielding a large value of χ^2 would be

- (a) the role assignments are not random, i.e., they show a tendency toward or away from complexity,
- or
- (b) the role assignments are not as they were assumed to be, i.e., they demonstrate a tendency toward or away from non-interchangeability among individual members or roles.

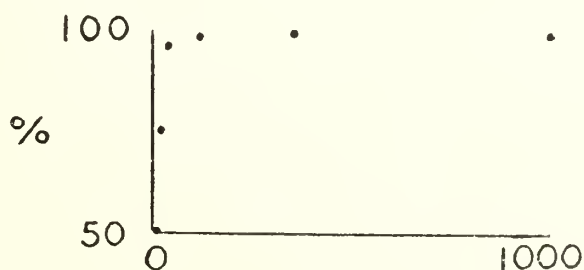
As an example (see Moser and Wyman, 1958), if a sample of 1000 organizations, all of size 10, were believed to fall under assumption IV, then $P(\underline{k}, 10) = \frac{\binom{9}{\underline{k}-1}}{2^9}$ for each \underline{k} . This is a binomial distribution with

$\underline{n} = 9$ and $p = .5$. We obtain a table as follows:

k	\hat{p}	P
1	0/100	.0020
2	0/100	.0176
3	0/100	.0703
4	0/100	.1641
5	0/100	.2461
6	1/100	.2461
7	2/100	.1641
8	7/100	.0703
9	23/100	.0176
10	67/100	.0020

This distribution would yield a $\chi^2_9 = 226.53$. A χ^2_9 value this large would occur less than one half of one per cent of the time by chance alone. The conclusion would be that the assignment of people to roles in this sample of organizations did not adhere to assumption IV.

A visual method of displaying deviations from predicted differentiation is to plot each observed percentile versus the number of organizations with a lower percentile. If the organization is differentiating in the predicted manner then the result should be a straight line from $(0,0)$ to $(\underline{n},1)$. The chi squared random variable measures the significance of any deviation from this straight line. The plot for the above example is:



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